Flutter Suppression in Long-Span Suspension Bridges by Arrays of Hysteretic Tuned Mass Dampers

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SUMMARY. Systems of nonlinear hysteretic tuned mass dampers are proposed toward multi-mode flutter mitigation in long-span suspension bridges. Systematic comparisons of the performance of the physically nonlinear absorbers with that of classical linearly viscoelastic absorbers are carried out. The equations of motion ensuing from a linearized parametric structural model of suspension bridges are coupled with the equations governing the dynamics of the control system and the time-dependent aerodynamic loads which are obtained through a quasi-steady nonlinear formulation. The equations of the controlled aeroelastic system are solved by reducing the PDE system according to the Galerkin method. Numerical simulations are performed to investigate the effectiveness of the vibration absorber system by comparing the bridge flutter condition and the post-flutter behavior with and without the control devices.

1 INTRODUCTION

Long-span suspension bridges are characterized by a low bending/torsional stiffness. Such flexibility and the high width-to-depth ratio, typical of the boxed, sharp-edge, deck cross sections usually adopted in the design of these structures, emphasize the effects of the aerodynamic loads arising from the wind-structure interaction. Thus, when subject to severe wind-induced excitations, suspension bridges may be affected by dynamic instability phenomena, such as flutter, that can overcome the in-service limit states and, in special cases, also to the collapse of the structure.

Flutter can be studied also in the context of linearized aeroelastic models which can provide, with reasonable accuracy, the range of wind speeds where the Hopf bifurcation of the equilibrium state occurs. However, these models are unable to describe some of the typical aeroelastic behaviors occurring at post-flutter speeds, and characteristic of nonlinear dynamical systems, such as limit cycle oscillations (LCOs). In particular, post-flutter behavior may be influenced by structural, geometric and constitutive, nonlinearities [1, 2, 3] and, more strongly, by the nonlinearities deriving from the aerodynamic loads [4, 5]. The latter are strictly related to the sharp-edge, bluff shape of the bridge deck and derives from viscous effects such as flow separation and reattachment around the section.

To account for these nonlinear effects, the quasi-steady aerodynamic theory, originally formulated for thin airfoil, has been successfully applied to the aeroelastic analysis of bridges. Within this formulation, the aeroelastic loads are evaluated through the aerodynamic coefficients defined as functions of the effective (dynamic) wind angle of attack. The validity of this approach is typically verified in a range of wind speeds where the dynamics of the bridge are slower with respect to the dynamics of the flow passing throughout the deck width. In this case, the so-called reduced velocity $U_r = U_w/(2\pi \omega B)$, where $U_w$ is the free-stream wind speed, $B$ is the bridge deck width, and $\omega$ is its circular frequency of oscillation in the flow, usually takes values greater than 10 ÷ 11. To consider the effects due to the unsteadiness of the relative motion between the section and the air flow, frequency-dependent or time-dependent indicial formulations can be adopted to predict more
accurately the critical wind speed at the onset of the flutter instability [6, 7, 8, 2, 3], although being inappropriate for dynamical investigations in post-critical speed regime. The design criteria for modern suspension bridges are today primarily devoted to the safety towards instability phenomena associated with wind-structure interaction. Within this context, new challenges deriving from the continuous increase of the span length (i.e., the Messina Straight Bridge may have a span of about 3300 m) arise from drastic reduction of the weight of the construction material and optimization of the performance of the structural elements. As a direct consequence, an increase of the bridges deck slenderness is experienced and with that the inability to guarantee that the flutter boundary be far enough for the safety of the structure for in-service wind speeds range.

Thus, an interesting design strategy could be that of accepting that the bridge might undergo the post-critical condition and mitigate its response by employing ad hoc vibration absorber systems. The passive architecture features arrays of devices already studied in [9, 10], where a visco-hysteretic absorber was theoretically studied and experimentally validated. In [10] the hysteretic absorbers were exploited to increase the flutter speed and improve the post-flutter behavior of airfoils. The hysteresis of such devices is described by the Bouc-Wen constitutive law modified to exhibit a hardening behavior.

Phenomena such as dynamic aeroelastic instabilities, including flutter and post-flutter, can be investigated effectively only in the context of a parametric modeling together with a continuum elastodynamic formulation which enables a straightforward investigation into the influence of the main design parameters on the critical and post-critical behavior of the bridge [11, 12]. In the technical literature several models of suspension bridges have been proposed, in particular, a linearized continuum formulation was first proposed in [13]. The equations of motion for suspension bridges were employed for aeroelastic investigations in [3, 14].

The analytical model here proposed is based on the Euler-Bernoulli beam theory for the 3D deck-girder dynamics, parametrized by one single space coordinate. On the other hand, the cables retain the fundamental stretching nonlinearity which is the cause of the nonsymmetric bridge behavior. This model is employed to study the dynamic Hopf bifurcation that occurs at the onset of flutter and the critical wind speed is evaluated by performing time-domain simulations. Moreover, the adoption of a quasi-steady nonlinear formulation of the aerodynamic loads allow the investigation of the post-flutter response and the optimization of a control strategy based on the use of linear and nonlinear tuned mass dampers. The feasibility of a passive control strategy based on linear tuned mass dampers for flutter control of suspension bridges was recently studied in [15]. The effects of the control system was investigated considering bridges exhibiting hard-type and soft-type flutter, respectively; these two behaviors, are representative of a rapid (hard-type) or slow (soft-type) decrease of the damping of the dynamical system, accounting for the structural damping and the negative contribution associated with the aerodynamic self-excited loads, occurring at wind speeds close to or higher than the flutter speed.

In the proposed work, an optimization technique is employed to determine the best spatial collocation and the optimal design parameters of a control architecture comprising an array of hysteretic vibration absorbers for increasing the flutter speed and for the mitigation of LCOs occurring at post-flutter wind speeds. Furthermore, comparisons between the aeroelastic responses of the bridge controlled by linear vibration absorbers or by hysteretic tuned mass dampers are proposed. The Runyang suspension bridge in China is considered as case study for which the nonlinear expressions of the aerodynamic coefficients for its deck cross-section calculated in [5] are adopted. The aeroelastic equations of motion of the bridge, together with the equations of motion of the hysteretic devices, are solved by discretizing the equations via the method of weighted residuals in the form due to
2 AEROELASTIC MODEL

A parametric model of suspension bridges subject to external time-dependent excitation is formulated. The Runyang Suspension Bridge, with a span \( l = 1490 \) m, is assumed as a suitable case study for the assessment of the control system. The deck width \( B \) is 35.9 m, the distance \( B_c \) between the suspension cables is 34.3 m; the equivalent area of the deck cross section is \( A_d = 1.2481 \) m\(^2\), the mass and the mass moment per unit length of the deck are \( m_d = 18387 \) kg/m and \( J_m^d = 1.852 \times 10^9 \) kg m\(^2\)/m, respectively; the equivalent Young modulus is \( E_d = 210 \) GPa. The flexural and torsional stiffness coefficients adopted in the calculations are \( k_d = 4.167 \times 10^{11} \) N m\(^2\) and \( k_b = 4.027 \times 10^{11} \) N m\(^2\), respectively. The mechanical characteristics of the suspension cables can be summarized as follows: \( E_c = 200 \) GPa, \( A_c = 0.47347 \) m\(^2\), \( m_c = 3817 \) kg/m, \( L_c = 1529 \) m and \( H_c = 2.41 \times 10^8 \) N, where \( L_c \) is the cables length and \( H_c \) the horizontal component of the cables tension induced by dead loads.

The boxed-type cross-section and the high flexural flexibility of the structure allow to neglect warping effects in the deck as well as shear deformations, thus, in the proposed linearized model, the girder-deck is considered as an Euler-Bernoulli beam. The deck bridge kinematics are assumed to be governed by infinitesimal displacement gradients, and the motion in the bridge span direction is neglected since there are no external loads acting along the bridge span-wise direction. Moreover, due to the high axial stiffness of the suspension hangers, they are assumed to be rigid and modeled as a uniform unstretchable membrane along the span, while the equilibrium of the cables under their weight is the catenary. The anchorage points of the cables atop the towers are considered fixed assuming the bridge towers sufficiently stiff to neglect any deflection of their tips. Since the flutter condition manifests itself through bending-torsional coupled oscillations, in the present work the assumption of negligible sway motion (deck width-wise direction) is made in consonance with the well-known fact that drag forces have a negligible effect on the critical flutter condition.

2.1 Equations of motion

The reference configuration of the bridge is represented in Fig. 2 where a fixed orthonormal basis \( (e_1,e_2,e_3) \) is introduced at the left end of the bridge with origin in the elastic center of the cross section. The direction \( e_2 \) denotes the bridge longitudinal axis and the \( (e_1,e_2) \)-plane represents the cross sectional plane, being \( e_2 \) the vertical axis. According to the convention adopted, \( u(x,t) \) denotes the vertical deflection of the deck centerline along the \( e_2 \) axis while \( \phi(x,t) \) denotes the torsional rotation about the longitudinal axis \( e_3 \).

To nondimensionalize time and space, respectively, the circular frequency \( \omega := \sqrt{k_d/(m_d \cdot l^2)} \) (\( k_d \) is the deck flexural stiffness in the vertical direction) and the bridge span \( l \) are adopted. The cable catenary configuration is represented by \( y_c(x) \) and depends on the mass per unit reference length supported by each cable, \( m_0 = (m_c + \frac{m_d}{A_d}) \), with \( m_c \) and \( m_d \) being the mass per unit length of each cable and of the bridge deck, respectively. Once the sag-to-span ratio \( d_c/l \) is assigned, the length of the cables can be calculated as \( L_c = l \int_0^l \sqrt{1 + y_c'^2} \, dx \) and its ratio with respect to the bridge span is \( L_c/l =: \eta_c \). The distance \( B_c \) between the two suspension cables and the deck width \( B \) are nondimensionalized with respect to \( l \), hence, \( b_c = B_c/l \) and \( b = B/l \); the nondimensional inertias are \( \bar{m}_d = (m_d + 2m_c)/m_d \), \( \bar{J}_m = \left( J_m^d + \frac{b^2}{2}\left( \frac{m_c}{m_d} \right) \right)/\left( (m_d \cdot l^2) \right) \). Moreover, the following stiffness ratios are introduced: \( \kappa_c = E_c A_c l^2/k_d \), \( \alpha_c = H_c l^2/k_d \), \( \beta = k_o/k_d \), where \( k_d \) is the deck flexural stiffness and \( k_o \) is the deck torsional stiffness. The nondimensional equations of motion

\[ \bar{m}_d \ddot{u} + k_d u = 0 \]
governing the bridge dynamics are thus obtained as

\[ \ddot{m}_d \ddot{u}_2 + \ddot{c}_2 \ddot{u}_2 + u_2''' - 2 \alpha_c u_2'' - 2 \kappa_c y_1' \frac{1}{\eta_c} \int_0^1 y_1' u_2' \, dx - \lambda_2 = 0, \]
\[ \ddot{J}_m \ddot{\phi}_3 + \ddot{c}_3 \ddot{\phi}_3 - \beta \phi_3'' - \frac{b_c^2}{2} \alpha_c \phi_3'' - \frac{b_c^2}{2} \kappa_c y_1'' \frac{1}{\eta_c} \int_0^1 y_1' \phi_3' \, dx - \lambda_3 = 0. \]  

(1)

The overdot denotes differentiation with respect to the nondimensional time whereas the prime represents the space- and time-dependent external force and torsional couple per unit reference length, respectively. Structural damping is introduced as a classical viscous damping, thus proportional to the centerline velocity. The ensuing nondimensional damping ratios are: \( \bar{c}_2 = 2 \xi \bar{m}_d \bar{\omega}_2 \) and \( \bar{c}_3 = 2 \xi \bar{J}_m \bar{\omega}_3 \). A damping factor \( \xi = 0.5\% \) is employed in the calculations and \( \bar{\omega}_2 \) and \( \bar{\omega}_3 \) are the nondimensional lowest natural frequencies of the corresponding vibrational modes of the bridge. A simply supported scheme in the plane \((e_2, e_3)\) and a hinged-hinged scheme in the plane \((e_3, e_1)\) are assumed for the bridge deck while the two suspension cables are supposed fixed atop the towers.

2.2 Aerodynamic loads

The aerodynamic loads, defined according to the nonlinear quasi-steady formulation, allow the system to be integrated in the time domain. These loads are represented by the following force and moment per reference length:

\[ \bar{F}_2(x, t) = L(x, t) \cos(\alpha_I) + \bar{D}(x, t) \sin(\alpha_I) \text{ and } \bar{M}(x, t). \]

\( \bar{F}_2 \) is the projection along the reference axis \( e_2 \) of the aerodynamic nondimensional lift \( \bar{L}(x, t) \) and drag \( \bar{D}(x, t) \). By assuming a zero initial wind angle of attack, the distributed aerodynamic forces strictly depend on the effective angle of attack \( \alpha_E \), and evaluated as follows:

\[ \alpha_E(x, t) = \phi_3(x, t) + \alpha_I(x, t) \]
\[ \alpha_I(x, t) = \frac{\ddot{u}_2(x, t) + \frac{b}{l} \phi_3(x, t)}{\bar{U} + \ddot{u}_1(x, t)} \]  

(2)

where \( \bar{U} \) is the nondimensional wind speed defined as \( \bar{U} = U_w / (l \bar{\omega}) \). The term \( \alpha_I(x, t) \) in Eq. 2 represents the instantaneous angle between the directions of the free-stream speed \( U_w \) and the relative velocity of the flow. Thus, the nonlinear expressions of the aerodynamic loads read:

\[ \bar{D}(x, t) = P \bar{b} C_D(\alpha_E), \quad L(x, t) = P \bar{b} C_L(\alpha_E), \quad \bar{M}(x, t) = P \bar{b}^2 C_M(\alpha_E), \]

(3)

where \( P \) is the nondimensional dynamic pressure, defined as \( P = \frac{1}{2} \rho U^2 \) and \( \rho = 1.225 \text{ kg/m}^3 \) is the air density. The nonlinear form of the static aerodynamic coefficient curves given by

\[ C_D(\alpha_e) = 0.041 + 0.131 \alpha_e + 3.962 \alpha_e^2 - 1.258 \alpha_e^3 - 11.399 \alpha_e^4, \]
\[ C_L(\alpha_e) = -0.005 + 6.097 \alpha_e - 0.209 \alpha_e^2 - 64.819 \alpha_e^3 - 6.698 \alpha_e^4 + 284.183 \alpha_e^5, \]
\[ C_M(\alpha_e) = 0.0119 + 1.052 \alpha_e - 0.395 \alpha_e^2 - 14.668 \alpha_e^3 + 2.248 \alpha_e^4 + 75.832 \alpha_e^5. \]

(4)

is shown in Fig. 4. The static lift, drag, and aerodynamic moment coefficients exhibit a nonlinear behavior already at moderate and high angles of attack. Forth- and fifth-order polynomial nonlinearities (see [5]) characterize the expression of the drag, the lift, and aerodynamic moment, respectively.

The flutter condition is obtained as solution of the eigenvalue problem which also delivers the flutter mode shape shown in Fig. 2.
Figure 1: Aerodynamic coefficients ($C_D$, $C_L$, $C_M$) for the Runyang deck section [5].

Figure 2: Reference configuration (light grey lines) and flutter mode shape (solid thick lines).

3 HYSTERETIC TUNED MASS DAMPERS
A passive nonlinear control system is introduced to mitigate the bridge vibrations induced by the wind-structure interaction. The control system consists of multiple arrays of nonlinear hysteretic tuned mass dampers, for which the constitutive law is described by the modified Bouc-Wen model. Multiple arrays of vibration absorbers are necessary to deal with feasible size and weight of the single devices (see Fig. 3). The absorbers are connected to the bridge deck on both sides so as to provide both point-wise control forces and control torques. Such architecture allows to control both vertical and torsional motions.
3.1 Control forces

Each vibration absorber within the arrays of absorbers contributes to the bridge dynamics with a control force and a control moment. The span-wise coordinate of the \(i\)-th device is denoted by \(x_i\) and its displacement, relative to the bridge deck, is \(y_i(t)\) so that the overall displacement is 
\[
Y_i(t) = y_i(t) + u_2(x_i, t) \pm (b_c/2)\phi_3(x_i, t).
\]
The \(\pm\) depends on whether the \(i\)-th TMD provides a positive or negative couple, respectively. In particular, the force and the moment exerted by the \(i\)-th TMD are defined as
\[
F_c^\pm = \sum_{i=1}^{N_{va}/2} \mu_i \left[ \ddot{u}_2(x_i, t) \pm \frac{b_c}{2} \phi_3(x_i, t) + \ddot{y}_i(t) \right] \delta(x - x_i), \tag{5}
\]
\[
M_c^\pm = \sum_{i=1}^{N_{va}/2} \pm \frac{b_c}{2} \mu_i \left[ \ddot{u}_2(x_i, t) \pm \frac{b_c}{2} \phi_3(x_i, t) + \ddot{y}_i(t) \right] \delta(x - x_i) \tag{6}
\]

where \(N_{va}\) is the overall number of vibration absorbers, \(\mu_i = M_i/(m_d l)\) is the nondimensional mass of each vibration absorber, \(\delta(x - x_i)\) is the Dirac-delta function, \(\ddot{u}_2(x_i, t)\) and \(\phi_3(x_i, t)\) are the deck vertical and torsional accelerations evaluated at the position of the \(i\)-th TMD whose relative acceleration is \(\ddot{y}_i\).

![Figure 3: Control architecture. Disposition of the TMDs along the bridge span and within the deck cross section.](image)

Each device is governed by a second-order differential equation defined as
\[
\mu_i \left[ \ddot{u}_2(x_i, t) \pm \frac{b_c}{2} \phi_3(x_i, t) + \ddot{y}_i(t) \right] + c_i \dot{y}_i(t) + k_i y_i(t) + z_i(t) = 0. \tag{7}
\]
The nondimensional hysteretic part of the constitutive TMD force, denoted by \(z_i(t)\), is governed by the well-known Bouc-Wen law. Typical values for \(c_i\) and \(k_i\) are drawn from the literature on TMD-based flutter control. Since the optimal TMD parameters were sought in the literature for linear TMDs only [15], here the hysteresis parameters are obtained through an iterative optimization procedure, based on the minimization of the overall deck centerline traveled distance evaluated at the first-quarter section.
Figure 4: Loci of the frequency and damping of the lowest symmetric and skew-symmetric modes of the uncontrolled (solid and dashed lines) and controlled (grey lines) bridge when the wind speed varies between 0 and 70 m/s.

4 NUMERICAL RESULTS

Numerical simulations in the frequency- and time-domain are carried out for the uncontrolled structure, to evaluate the flutter speed and the associated flutter mode shape. The eigenvalue problem is first investigated by increasing the value of the wind speed so as to determine the critical condition (see Fig. 4). In the case study analyzed, the flutter speed is attained at a wind speed equal to 52 m/s and the flutter mode shape is the first skew-symmetric bending-torsional mode. In particular, the second flutter mode shape is attained for a wind speed slightly higher (see blue line in Fig. 4). The vicinity of the lowest two flutter conditions implies that the control architecture must account also for the second flutter condition. After identifying the flutter modes, the control architecture can be defined by placing the devices where the control action is maximized.

For control of the first mode, a cluster of 5 pairs of TMDs is distributed about the first quarter-span section and a second cluster is placed about the second quarter-span section. Moreover, due to the proximity of the second critical condition to the first critical condition (see Fig. 4), to control also the symmetric mode, a third cluster of TMDs is located about the mid-span section. Thus the control architecture becomes a multi-mode flutter control system and the optimal parameters of the TMDs are evaluated for two different frequencies, whose values are obtained from Fig. 4, accounting for the variations of natural frequencies of the structure due to the interaction with the aerodynamic loads. By introducing an equivalent array of multiple visco-elastic TMDs, the flutter speed increases up to 10% (see Fig. 4). This result is in agreement with the literature [15]. The use of passive TMDs does not allow to shift the flutter condition considerably for hard-type flutter such as that exhibited by typical suspension bridges, thus is of interest to investigate the post-flutter behavior of the controlled structure. The analyses are performed in the time domain by varying the wind speed and the amplitude value of the limit cycle oscillations is evaluated in the considered range of wind speeds; the results are shown in Fig. 5. The response of the uncontrolled structure (solid lines) shows the typical super-critical behavior of a Hopf bifurcation, in which the amplitude of the LCO grows rapidly with the wind speed. On the other hand, the controlled structure endowed with visco-elastic TMDs (dashed lines) shows its bifurcation point at a wind speed value higher than the flutter speed of the uncontrolled structure. Moreover, the amplitude of the LCO is lower than that of the uncontrolled bridge when the linear control system is in action. When the nonlinear hysteretic
Figure 5: Bifurcation diagrams showing the LCO amplitude (deflection and torsional angle at $l/4$) past the flutter condition when the wind speed varies between 50 and 70 m/s.

Figure 6: Time histories at two wind speeds: $U_w = 57$ m/s (top) and $U_w = 65$ m/s (bottom). Response of the uncontrolled (black solid lines) bridge, controlled by linear TMDs (light grey lines) and controlled by nonlinear TMDs (dark grey lines).
absorbers are adopted (grey solid lines), the critical condition is attained at a wind speed value close to that of the uncontrolled case, but the slope of the bifurcated branch is much smaller. In particular, at the wind speed value of 65 m/s the amplitude of the LCO is 5 times smaller. To better appreciate this finding, the responses of the uncontrolled and controlled structure are evaluated for two wind speeds, \( U_w = 57 \) m/s and \( U_w = 65 \) m/s. The time histories at the first quarter-span section are shown in Fig. 6. The LCO amplitude of the structure controlled by the proposed nonlinear absorbers is much lower than both the uncontrolled response and the response controlled by linear TMDs.

5 CONCLUSIONS

Wind-excited long-span suspension bridges often exhibit hard-type flutter which may also involve two close flutter modes as shown in the context of the Runyang Suspension Bridge with a span of 1490 m. Previous studies have demonstrated that, for bridges suffering hard-type flutter, the insertion of linear tuned mass dampers can shift the onset of flutter to wind speeds which are only slightly higher than the flutter speed of the uncontrolled structure. This is due to the fact that the additional damping introduced by the TMDs cannot win the negative damping induced by the self-excited aeroelastic loads (negative aeroelastic damping grows at high rates with the wind speed) but for a narrow range of wind speeds past the flutter condition of the uncontrolled structure.

New bridge designs are currently exploring super-long spans and more aerodynamic cross-sectional shapes which may entail (multi-mode) flutter at lower wind speeds. It is likely that the flutter condition may turn out to be one of the severe limit states to deal with. This motivates the development of new flutter control strategies. The original aspect of the present work is the proof that the insertion of hysteretic hardening tuned mass dampers, in spite of the lack of authority in shifting the flutter condition appreciably, can effectively control the post-flutter response reducing the amplitude of the LCOs by orders of magnitudes. The obtained results are based on a parametric model for the suspension bridge incorporating a quasi-steady nonlinear formulation for the aerodynamic loads together with the equations of motion for the clusters of absorbers exhibiting hardening hysteresis. Optimization tools have been employed to achieve an optimal design of the hysteretic flutter absorbers.

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